1. Work out $\frac{3}{2} - \frac{1}{6}$ as both a mixed number and a improper fraction.

1) $\frac{3}{2} - \frac{1}{6} = \left(\frac{3}{2} \times \frac{3}{3}\right) - \frac{1}{6}$

$= \frac{9}{6} - \frac{1}{6} = \frac{8}{6} = 1\frac{1}{3}$

2. Work out $64^{\frac{1}{2}} \times 8^{\frac{1}{3}}$

$64^{\frac{1}{2}} \times 8^{\frac{1}{3}} = \sqrt{64} \times \sqrt[3]{8}$

$= 8 \times 2$ since $8 \times 8 = 64$ and $2 \times 2 \times 2 = 8$

3. Simplify, $\frac{1+\sqrt{3}}{3+2\sqrt{3}}$

$$\frac{1+\sqrt{3}}{3+2\sqrt{3}} = \frac{1+\sqrt{3}}{3+2\sqrt{3}} \times \frac{3-2\sqrt{3}}{3-2\sqrt{3}} = \frac{3-6+3\sqrt{3}-2\sqrt{3}}{9-12}$$

$$= \frac{-3+\sqrt{3}}{-3}$$

$$= \frac{3-\sqrt{3}}{3}$$

4. Work out $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

$$\left(\frac{8}{27}\right)^{-\frac{2}{3}} = \left(\frac{8}{27}\right)^{-1\times 2\times \frac{1}{3}}$$

$$= \left(\left(\frac{8}{27}\right)^{-1}\right)^{2\times\frac{1}{3}} = \left(\frac{27}{8}\right)^{2\times\frac{1}{3}}$$

$$= \left(\left(\frac{27}{8}\right)^{\frac{1}{3}}\right)^{2}$$

$$= \left(\frac{3}{2}\right)^{2} = \frac{9}{4}.$$

Challenge

1. Calculate the prime factorisation of 24 and of 16
2. Calculate $lcm(24, 36)$ and $hcf(24, 36)$
3. Verify that $lcm(24, 36) \times hcf(24, 36) = 24 \times 36$
4. Think about why $lcm(x, y) \times hcf(x, y) = x \times y$

$24 = 3 \times 8 = 3 \times 2^3 \quad 16 = 2^4$

- For lcm, pick the bigger power,

2^3 vs 2^4 pick 2^4

and 3^1 vs 3^1 pick 3^1 so $lcm(24,36) = 3 \times 2^4$

- For HCF, pick the lower power,

so we pick 2^3 and 3^1 so

$hcf(24,36) = 2^3 \times 3$

then $hcf(24,36) \times lcm(24,36) = 2^3 \times 2^4 \times 3 \times 3$

$= 24 \times 36$

Slightly difficult.

- For $x = a^{p_1} \times b^{p_2} \times \cdots$ written as prime decomposition,

and $y = a^{q_1} \times b^{q_2} \times \cdots$

then $lcm(x,y) = a^{max(p_1,q_1)} \times b^{max(p_1,q_1)} \times \cdots$

and $hcf(x,y) = a^{min(p_1,q_1)} \times b^{min(p_1,q_1)} \times \cdots$

So,

$$lcm(x,y) \times hcf(x,y) = a^{max(p_1,q_1)+min(p_1,q_1)} \times b^{min(p_1,q_1)+max(p_1,q_1)} \times \cdots$$

$$= a^{p_1+q_1} \times b^{p_1+q_1} \cdots\cdots$$

$$= x \times y.$$

since $min(a,b) + max(a,b) = a+b$ and using that $x^{a+b} = x^a \times x^b$.